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Delta Operator modeling and Control using Genetic Algorithm Controllers Based on Ga and De

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ABSTRACT

The main objective is to apply the genetic algorithm to solve different control problems in the complex domain. The problems of single input single output i.e. PID controller design, 1st order system, 2nd order system, delay system, Multi input and single output system and multi input multi output system are in continuous-time systems. The scalar performance index as an error function between the developed reference model and the closed loop system, where the controller parameters are computed to minimize the performance index. The genetic algorithm and differential evolution has been used as an optimization tool to minimize a performance index (fitness function) that has been defined for each of the above problems.

1. Introduction:

The rapid advancement of science and technology and the advent of low cost, reliable and high speed digital computers have led to extensive research in the area of computer aided control system analysis and design. The aim is to solve some problems in identification, modeling and controller design by applying the genetic algorithm.

Genetic algorithms (GA) are search procedures inspired by the laws of natural selection and genetics. They can be viewed as a general-purpose optimization method and have been successfully applied to search, optimization and machine learning tasks. GA has the ability to solve difficult, multi dimensional problems with little problem-specific information and hence has been chosen as the optimization technique to solve various problems in control systems.

The mathematical procedures of modeling practical systems lead to descriptions of the process in the form of complex high -order transfer functions or states space models. These high-order models are difficult to use for simulation, analysis or controller synthesis, and it is not only desirable but also often necessary to obtain satisfactory reduced order representations of such high order models. A need exists for design methods that may be used to arrive at simple low-order implement able controllers that can adequately control plants or processes regardless of their order, complexity or stability.

Often the need arises to implement a digital controller in place of an existing analog controller. One may redesign a new controller in the digital domain or may discretize the existing continuous-time controller.

Advanced controller design techniques such as LQG, H_∞ etc., often lead to controllers whose order may be equal to or even exceed the order of the plant. A suitable low-order controller that retains the dominant characteristics of the original high-order controller is desired in such cases.

2. Continuous-Time Controller Design:

A new GA based method is proposed for the design of rational continuous-time controllers for linear time-invariant single-input single-output (SISO) and multi-input multi-output (MIMO) systems. The design method is comprehensive in nature and is applicable to a wide range of plant models. Selection of an appropriate reference model extends the usefulness of the method to controller design for unstable systems and non-minimum-phase plants. The method relies on the concept of approximate model matching and yields implement able low-order rational controllers using only output feedback. Increase in the computational burden with increase in the order of the plant model is negligible thus obviating the need for order reduction of the plant transfer function. The validity of the method is illustrated by some examples from the literature.

The problem of model matching control consists of designing a controller to compensate a given plant so that the resultant controlled system has a pre-specified transfer function (reference model). The obvious advantage of this approach is that the design specifications (time and frequency domain) that are implicated through the reference model will be met by the controlled system. Design techniques for exact model matching as well as approximate model matching have been proposed for systems described by both state-space and transfer function models [Chen (1970)]. Exact model matching, however, is known to bring about certain practical difficulties. It requires overly complicated implementations. System configurations typically involve both feed forward and feedback compensation [Chen (1987)], in addition, the compensator TFs are generally of roughly the same order as the plant TF. Thus, for high-order plants, the controller implementation becomes impractical.The design of a control system is formulated as an approximate model -matching problem. That this problem is of great practical importance and provides a viable alternative for the design of effective, implement able controllers is evident from the number of publications in this area (given above). The objective of an approximate model matching type of control system design may be stated as: Given an open loop plant TF, P(s), design an overall closed-loop system so that the plant output y(t) will follow a given reference input as closely as possible. The design highly depends on the chosen reference input. Throughout the paper, we consider a step input. The closeness of the plant output tracking the reference step input is checked in terms of steady state and transient performances. In the steady state, we require the steady-state error to be zero and for good transient performance, we require a prescribed rise time, settling time, and overshoot. We also consider frequency-domain characteristics such as the gain and phase margins, bandwidth, cut-off rate etc. implicated through the reference TF as the desired design objectives. Thus, the controller design problem becomes: Given a plant and a reference input (step input), design an implement able dynamic controller that uses only output feedback and yields an overall closed-loop system to meet a given set of steady-state and transient performance criteria. One of the important aspects of controller design and implementation is the order of the controller and the subsequent hardware complexity. Practicing engineers prefer implement able controllers of low complexity. Various design methods have been proposed to obtain low-order compensators based on the Padé approximation technique Pal (1993), continued fraction expansion Chen (1970), least squares minimization Belanger (1976), and model reduction techniques Lepschy (1985) etc. Kreisselmeier and Mevenkamp (1988) proposed a method which involves an a-posteriori refinement of the controller that has been synthesized using a reduced-order model of the plant. In the model matching method of Sanathanan and Quinn (1987), frequency domain optimization is used to obtain a low-order approximate of the ideal controller given by the synthesis equation [D'Azzo (1981)]. In the proposed method, the controller parameters are obtained by minimizing a performance index (fitness function) using the GA and use the concept of approximate model matching. Selection of an appropriate reference model extends the usefulness of the method to controller design for unstable systems and non-minimum-phase (NMP) plants.

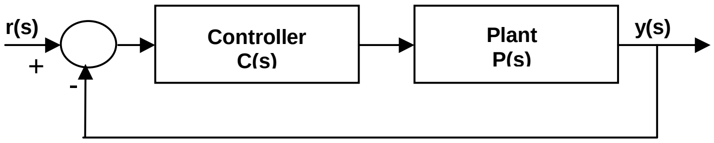


Fig: 1 Standard unity feedback configuration

The Exact Model Matching Problem

Consider the continuous-time unity feedback control system configuration as shown in Fig. (2.1).P(s) and C(s) are respectively the plant and controller transfer functions and are given by

$$P_{m,n}(s) = \frac{\sum_{i=0}^{m} b_i s^i}{\sum_{i=0}^{n} a_i s^i}, \quad m \le n \qquad (2.1)$$

$$C_{p,q}(s) = \frac{\sum_{i=0}^{p} \beta_i s^i}{\sum_{i=0}^{q} \alpha_i s^i}, \quad p \le q \qquad (2.2)$$

The subscripts on the left-hand side of Eqn. (2.1) and Eqn. (2.2) indicate the orders of the numerator and denominator respectively. The closed loop transfer function F(s) is then given by

$$F_{m+p,n+q}(s) = \frac{\left(\sum_{i=0}^{m} b_i s^i \sum_{i=0}^{p} \beta_i s^i)\right)}{\left[\sum_{i=0}^{n} a_i s^i \sum_{i=0}^{q} \alpha_i s^i + \sum_{i=0}^{m} b_i s^i \sum_{i=0}^{p} \beta_i s^i\right]} \qquad (2.3)$$

The denominator of Eqn. (3.3) represents the characteristic polynomial of the closed-loop system and is of order (n+q). The unknowns in Eqns. (2.2) and (2.3) are the β_is and α_is corresponding to the compensator C(s).

In the exact model-matching problem, it is desired to find the unknown parameters β_i and α_i of C(s) such that closed-loop TF, F(s), exactly matches a general specification TF, M(s), given by

$$M_{k,l}(s) = \frac{\sum_{i=0}^{k} d_i s^i}{\sum_{i=0}^{l} c_i s^i}, \quad k \le l \qquad (2.4)$$

Thus, for exact model matching, we have

$$F(s) = M(s)$$

$$or \quad \frac{P(s)C(s)}{[1 + P(s)C(s)]} = M(s) \qquad (2.5)$$

Solving for C(s), we get

$$C(s) = \frac{M(s)}{[P(s)\{1 - M(s)\}]} \qquad (2.6)$$

Substituting for P(s) and M(s) from Eqns. (2.1) and (2.4), we finally have

$$C(s) = C_{k+n,m+l}(s) = \frac{\sum_{i=0}^{k} d_i s^i \sum_{i=0}^{n} a_i s^i}{\left[\sum_{i=0}^{m} b_i s^i \sum_{i=0}^{l} c_i s^i - \sum_{i=0}^{m} b_i s^i \sum_{i=0}^{k} d_i s^i\right]} \quad (2.7)$$

This is also called the 'Synthesis Equation' or 'Truxal's method' for designing C(s) [D'Azzo and Houpis (1981)]. This controller has been termed as the 'Ideal Controller'.

In some cases, it is found that this design method may not lead to the simplest form of compensator. The resulting controller may be of high-order and/or unstable. Moreover C(s) may not be realizable, i.e. improper. Further, the structure and order of the controller C(s) cannot be fixed a-priori as has been done in Eqn. (2.2). In the case of Approximate Model Matching (AMM), Eqn. (2.5) is only approximately satisfied, i.e.

$$F(s) \approx M(s) \quad (2.8)$$

Reference Model Selection: In the model-matching type of controller design followed in this thesis, the design goals are specified at the outset in the form of a reference model TF. The structure and complexity of the controller depends on the choice of the reference model TF. The reference model TF should be chosen to have a sufficiently rapid response; on the other hand, it should keep the high frequency gain of the controller small to avoid saturation in the actuators. The reference model might be required to satisfy some of the following design specifications.

The time-domain specifications, e.g., the rise time, overshoot, settling time and steady state error.

The frequency-domain specifications, e.g., the bandwidth, cut-off rate, gain margin and phase margin.

The complex-domain specifications, e.g., the damping ratio, damping factor, undamped natural frequency.

Quadratic optimal criterion.

Some representative model selection procedures are briefly given below.

Method: 1. Let a model TF be specified as

$$M(s) = \frac{d_0 + d_1 s}{c_0 + c_1 s + s^2}$$

for steady-state matching, $d_0=c_0$. Then

$$M(s) = \frac{c_0 + d_1 s}{c_0 + c_1 s + s^2} \quad (2.9)$$

Let the desired closed-loop specifications that M(s) has to satisfy be

Velocity error constant: k_v

Crossover frequency: ω_c

Damping ratio: ξ

The second-order model M(s) of Eqn. (2.9) may be chosen to satisfy these specifications. The parameters c_0, c_1 and d_1 are determined by solving the following equations [Chen (1970)].

$$\omega_c^2 c_1^2 - 2\omega_c^2 c_1 d_1 - c_0^2 = -\omega_c^4$$
$$c_1 - \left(\frac{c_0}{k_v}\right) - d_1 = 0$$
$$c_1^2 - 4c_0\xi^2 = 0 \quad (2.10)$$

For example, when k_v=20, ω_c=4.5 and ξ=0.785, Eqn. (3.10) gives,

$$M(s) = \frac{8.009 + 4.04265s}{8.009 + 4.4431s + s^2}$$

Method: 2. Let the desired specifications be

Maximum overshoot: 10%

Time to first peak: 0.01s

Unity final value: no steady-state error,

Choosing M(s)

$$\frac{\omega_0^2}{s^2 + 2\xi\omega_n s + \omega_n^2},$$

and following the design rules of thumb of Shieh (1981);

$$\omega_n \approx \frac{\pi}{4} \approx 300\,\text{rad/s}, \quad M_p \approx \exp(-\xi\pi) \rightarrow \xi = 0.75$$

Then,

$$M(s) = \frac{90000}{90000 + 450s + s^2}$$

Method: 3. A reference model M(s) that is optimal in the sense of minimizing a quadratic performance index may be chosen as given in Chi-Tsong (1990). For example, an optimal system TF may be chosen to minimize the quadratic performance index

$$J = \int_0^\infty \left[q(y(t) - r(t))^2 + pu^2(t)\right]dt, \quad \text{where } q, p > 0.$$

A certain value for q and p may be initially chosen and M(s) may be found out. If this model does not meet other requirements like control effort, then a different value each is chosen for q and p and the steps repeated to find another suitable M(s).

Method: 4. A reference TF

$$M(s) = \frac{1}{(1+sT)^2}$$

may be chosen Sanathanan (1987) and Quinn (1990). Certain features of this TF models are

It has only real poles,

It contains no numerator dynamics beyond those dictated by the loop type,

It gives sufficient response speed,

Reasonable robustness (in terms of gain and phase margins that can be imbedded in the TF model), It gives reasonable idea of the closed-loop time responses,

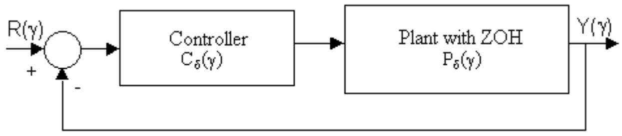
Location of the real poles may be determined by the system specifications for bandwidth and response time.

2 Controller Design in Complex Delta Domain – SISO

Introduction: With the present day availability of very high speed digital hardware which are less expensive and more reliable, digital control of systems using high speed digital computer has become popular. Consequently, a need is felt for methods of computer-aided digital controller design which are devoid of complex mathematical operations, are conceptually elegant and lead to low-order controllers while satisfying the desired specifications. In this Chapter, we propose one frequency domain techniques that satisfy all these goals.

The proposed frequency domain methods are based on the principle of approximate model matching. The desired specifications are translated into a target closed-loop transfer function, called the reference model. The purpose is to design a digital controller such that the closed-loop system matches the reference model in some sense. The method is proposed to design controllers in the frequency domain, is controller design by matching Delta Time Moments (DTMs), The sole objective of the proposed method is to design digital controllers such that the frequency response of the resulting closed-loop system matches a specified frequency response. Several design techniques for exact and approximate model matching have been proposed for continuous-time systems described by both state-space and transfer function models. For discrete-time system in shift operator parameterization, the frequency domain controller design methods vis-a-vis the complex curve fitting (CCF) method of Rattan and Yeh, dominant data matching (DDM) method of Shieh et al, and the simplex optimization (SIM) of Shi and Gibbard are worth mentioning. In the method of Rattan and Yeh, the frequency responses are matched to give a minimum weighted mean-square error. The method requires the evaluation of definite integrals and it is known that the stability of the resultant system depends on the upper limit of the chosen frequency range for optimization. The method of Shieh et al relies on analysis of the frequency response of the system and the choice of the critical frequencies called the dominant data is much dependent on the experience and judgment of the designer. The method may lead to non-linear algebraic equations, the solution of which is known to be sensitive to the initial guess vector and may often converge to a local minima. The method of Shi and Gibbard minimizes a sum of the squared error function in the magnitude and phase of the frequency response deviations while imposing constraints on the gain and pole-zero locations of the controller. The simplex optimization method is used and a detailed root locus analysis is required to search for the desirable stable locations of the poles and zeros of the controller in the unit circle in the complex z-plane. The computational burden of this method is heavy and the final solution is sensitive to the initial guess values. Pal, has proposed a method that uses the bilinear transformation and the principle of approximate model matching in the Padé sense.

We discuss here three unified controller design methods for delta operator parameterization of the discrete-time system in δ-domain. The proposed method is computationally simple and requires only the solution of a set of linear algebraic equations. Detailed a-priori study of the frequency response is not required to arrive at the equations and no iterative optimization techniques are involved. The computational algorithms are numerically stable and converge to a continuous-time like controller at very fast sampling.



The standard unity negative feedback configuration

Exact model matching problem

Consider the co-operator parameterized discrete-time unity feedback system configuration as shown in Figure (2.1). $P_\delta(\gamma)$ and $C_\delta(\gamma)$ are respectively the plant and controller transfer functions and are given by

$$P_{\delta(m,n)}(\gamma) = \frac{\sum_{i=0}^{m} b_i\gamma^i}{\sum_{i=0}^{n} a_i\gamma^i}; \qquad m \le n \quad ----2.1$$

$$C_{\delta(m,n)}(\gamma) = \frac{\sum_{i=0}^{p} \beta_i\gamma^i}{\sum_{i=0}^{q} \alpha_i\gamma^i}; \qquad p \le q ----2.2$$

The subscripts on the left-hand side of equations (2.1) and (2.2) represent the order of the numerator and denominator respectively. The closed-loop transfer function $G_\delta(\gamma)$ is then given by

$$G_{\delta(m+p,n+q)}(\gamma) = \left[\frac{\left(\sum_{i=0}^{m} b_i\gamma^i \sum_{i=0}^{p} \beta_i\gamma^i\right)}{\sum_{i=0}^{n} a_i\gamma^i \sum_{i=0}^{q} \alpha_i\gamma^i + \sum_{i=0}^{m} b_i\gamma^i \sum_{i=0}^{p} \beta_i\gamma^i}\right] -----2.3$$

The denominator of equation (2.3) represents the characteristic polynomial of the closed-loop system and is of order (n + q). The unknowns of equation (2.2 are the $\beta_i s$ and $\alpha_i s$ corresponding to the compensator $C_\delta(\gamma)$. In the exact model matching problem, it is desired to find the unknown parameters $\beta_i s$ and $\alpha_i s$ of $C_\delta(\gamma)$,such that the closed-loop transfer function, $G_\delta(\gamma)$ exactly matches a general specification TF, $M_\delta(\gamma)$, given by

$$M_{\delta(k,l)}(\gamma) = \frac{\sum_{i=0}^{k} d_i\gamma^i}{\sum_{i=0}^{l} c_i\gamma^i}; \qquad k \le l ---2.4$$

Therefore, for exact model matching, we have

$G_\delta(\gamma) = M_\delta(\gamma)$ ------2.5

$$\frac{p_\delta(\gamma)C_\delta(\gamma)}{1+P_\delta(\gamma)C_\delta(\gamma)} = M_\delta(\gamma) --- 2.5(a)$$

Solving for $C_\delta(\gamma)$,we have

$$C_\delta(\gamma) = \frac{M_\delta(\gamma)}{P_\delta(\gamma)[1-M_\delta(\gamma)]}. --- 2.6$$

Substituting for $P_\delta(\gamma)$ and $M_\delta(\gamma)$ from equations (2.1) and (2.4), we finally have

$$C_\delta(\gamma) = C_{\delta(k+n,m+l)}(\gamma) = \frac{\left(\sum_{i=0}^{k} d_i\gamma^i \sum_{i=0}^{n} a_i\gamma^i\right)}{\left[\sum_{i=0}^{m} b_i\gamma^i \sum_{i=0}^{l} c_i\gamma^i - \sum_{i=0}^{m} b_i\gamma^i \sum_{i=0}^{k} d_i\gamma^i\right]} \text{-2.7}$$

This is the so called "Synthesis equation" or "Truxal's method" for controller synthesis. The drawbacks of the above design are that the resultant controller may not lead to the simplest form; in addition, the controller may be of high order and or /unstable. Moreover the controller TF may not be realizable, i.e. it may be improper. Further, the structure and order of the controller cannot be fixed a-priori as has been done in equation (2.2). With this background knowledge, the approximate model matching (AMM) method is proposed for controller design of δ-operator systems. Accordingly, equation (2.5) is only approximately satisfied, i.e.

$$G_\delta(\gamma) \approx M_\delta(\gamma) \text{ ---2.8}$$

In the next section the AMM method for controller design in the δ-domain is discussed. Using the concept of AMM, the difficulties encountered in Exact Model Matching (EMM) method can be effectively resolved. It is further possible to design a compensator of chosen order and structure that approximately satisfies the various specifications embodied in the desired TF, $M_\delta(\gamma)$. The method provides added flexibility of making a tradeoff between the order/complexity of the controller and the extent to which the desired specifications are met. For $C_\delta(\gamma)$ to be physically realizable,

$$\begin{aligned}(m+l) &\geq (k+n)\\ (l+k) &\geq (n-m)\end{aligned} \quad 2.9$$

i.e $M_\delta(\gamma)$ must be selected so that the excess of (finite) poles over (finite) zeros for the closed-loop function is at least equal to the pole-zero excess of the plant TF, $P_\delta(\gamma)$. Therefore to make both the EMM and AMM methods to be feasible, the above degree constraints on the choice of $M_\delta(\gamma)$ must be imposed.

The approximate model matching problem:

The Approximate Model Matching (AMM) methods proposed here are in wide use in the area of reduced order modeling of continuous-time system and also discrete-time systems in shift operator parameterization. In the frequency domain, we will use the reduced order modeling philosophy for approximate model matching. Model order reduction seeks to find an approximate low-order TF, $M_\delta(\gamma)$, from a high order TF, $G_\delta(\gamma)$, such that equation (5.8) is satisfied, i.e. the two function are made approximately equivalent in some sense. In the DTM technique, few proportional DTMs of the respective systems are made identical, i.e.

$$\left.\frac{d^i G_\delta(\gamma)}{d\gamma^i}\right|_{\gamma=0} = \left.\frac{d^i M_\delta(\gamma)}{d\gamma^i}\right|_{\gamma=0} --- 2.10$$

I=0,1,2,……………….,(k+l+1)

$$g_{\delta i} = m_{\delta i} \qquad --- 2.11$$

In the area of reduced order modeling, it is known that if the initial few time moments of two systems are identical, their low-frequency responses would come close. The same philosophy is used here for δ-operator parameterized systems, and since most control systems are low pass, it is expected that the controllers designed by matching DTMs would ensure that the low frequency response of the controlled system is close to the desired one.

Selection of standard reference model

Approximate model matching type controller design methodology aim to find a control law such that the input-output description of the augmented system and a specification model are approximately equivalent. The success of the model matching type of controller design greatly depends on the development of a reference model which incorporates all the time, frequency and complex domain design specifications. For discrete-time systems in δ-operator representation we discuss in details the time and frequency domain specifications used for reference model selection.

Control system specifications in δ-domain: For a discrete-time system in δ-domain having an order higher than two, relations between the specifications in the time, frequency and complex δ -domain can be very complicated. In many cases, however, the dynamic characteristics of high-order control systems are well represented by those of a second order system or model for which the relationships between specifications are simpler. The second order transfer function

$$M_\delta(\gamma) = \frac{A\gamma + B}{\gamma^2 + C\gamma + D} --- 2.12$$

is chosen as the model for the closed loop discrete-time system in δ-domain. For a pole-zero form of this transfer function in z-domain, Kuo, has derived expressions for the time domain specifications, in terms of a set of complex z-domain specifications. Jury, has developed relationships between the system frequency response and its time response; however Shi and Gibbard have related both time and frequency domain specifications with the complex z-domain specifications. In δ-domain, however, no exhaustive study has been made so far to relate time and frequency domain specifications with the complex δ-domain specifications. An attempt is made to address these issues.

Second order reference model in δ-domain: A second order reference model in δ-domain is considered. The location of the complex pole $(p_\delta, p_\delta^{'})$ *and* the real zero $z_{\delta 1}$ of the delta transfer function $M_\delta(\gamma)$ are shown in Figure 2.2.

The zero is arbitrarily assigned along the real axis by the angle ρ as shown in Figure 2.2. Assume that the specifications for the closed-loop system performance are expressed in the complex γ-plane in terms of damping ratio ξ, the undamped natural frequency ω_n and the angle ρ. The purpose is to express the *A, B, C* and *D* coefficients of the closed-loop transfer function of eqn.(2.12) in terms of these parameters. The second-order discrete system with unity feedback, as in eqn.(2.12) is adopted for analysis. If it's open-loop transfer function is $F_\delta(\gamma)$, the closed-loop transfer function can be expressed in the following form:

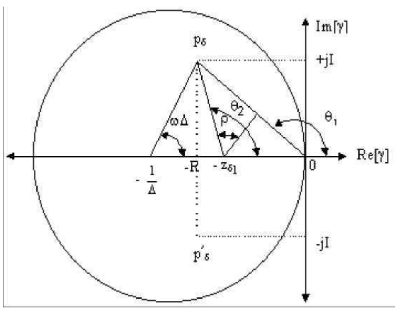


Figure: 2.2 Location of poles and zeros of the Reference model

$$M_\delta(\gamma) = \frac{F_\delta(\gamma)}{1+F_\delta(\gamma)} ---2.13$$

$$M_\delta(\gamma) = \frac{A(\gamma + z_{\delta 1})}{(\gamma - \rho\delta)(\gamma - \rho_\delta^{'})} ---2.14$$

$$M_\delta(\gamma) = \frac{A(\gamma + z_{\delta 1})}{[\gamma - (-\frac{1-e^{-\sigma\Delta + jw\Delta)}}{\Delta}][\gamma - (\frac{1-e^{-\sigma\Delta + jw\Delta}}{\Delta})} ---2.15$$

where ω is the damped natural frequency of system oscillation (rad./sec.) and is related to ω_n (natural frequency) by

$$\omega = \omega n\sqrt{1-\xi^2} ---2.16$$

Δ is the sampling period (s), and is related to the sampling frequency ω_s (rad./sec.) by

$$\Delta = \frac{2\Pi}{\omega_s} ----2.17$$

If we denote the real and imaginary parts of p_δ as *R* and *I*, respectively, the poles may be expressed as

$$p_\delta = -R + jI ----2.18$$

$$p_\delta^{'} = -r - jI ---2.19$$

where

$$R = \frac{1-e^{\sigma\delta}\cos w\Delta}{\Delta} ---2.20$$

$$I = \frac{e^{\sigma\delta}\sin w\Delta}{\Delta} ---2.21$$

From the geometry of Figure 2.2, we can write,

$$\rho = \theta_2 - \theta_1 + \frac{\pi}{2} ---2.22$$

Where

$$\theta_1 = \tan^{-1}(\frac{1}{-R}) ---2.23$$

$$\theta_2 = \tan^{-1}(\frac{1}{z\delta_2 - R}) ---2.24$$

From eqn. (3.21),

$$\tan(\rho - 90^0) = \tan(\theta_2 - \theta_1) --- 2.25$$

Expanding the right-hand side, and substituting for $\tan\theta_1$ and $\tan\theta_2$ from eqns. (3.24) and (5.25), we obtain the real zero as

$$z_{\delta 1} = \frac{R\tan(\rho - \frac{\pi}{2})(R^2 + I^2)}{[R\tan(\rho - \frac{\pi}{2}) - 1]} ---2.26$$

Note that $z_{\delta 1}$ is permitted to lie in the range $(-\infty, 0)$ on the real axis, ρ will vary from a lower limit $(\rho_1)\ to \frac{\pi}{2}$

The lower limit ρ_1 of ρ is

$$\rho_1 = \lim_{z_{\delta 1} \to -\propto} \rho = \lim_{z_{\delta 1} \to -\propto} (\theta_2 - \theta_1 + \frac{\pi}{2}) -----2.26$$

substituting eqns.(2.23) and (2.24) yields

$$\rho_1 = \frac{\pi}{2} - tan^{-1}(\frac{I}{-R}) ----5.27$$

From eqns.(2.16) and (2.19)

$$C = 2\frac{1-e^{-\sigma\Delta}cos\omega\Delta}{\Delta} -------2.28$$

$$D = [(\frac{1-e^{-\sigma\Delta}cos\omega\Delta}{\Delta})^2 + (\frac{e^{-\sigma\Delta}sin\omega\Delta}{\Delta})^2] -----2.29$$

The gain in eqn. (3.18) affects only the steady state response of the closed-loop system. It is assumed that under steady state conditions the difference between the output and input signals is zero. Hence from eqns. (2.12) and (2.14)

$$\frac{Y(\gamma)}{U(\gamma)} = \left|\frac{A\gamma + B}{\gamma^2 + C\gamma + D}\right|\gamma = 0 = \frac{B}{D} = 1 -----2.30$$

B=D

And

$\frac{Az_{\delta 1}}{p_{\delta p'_{\delta}}} = 1$-----2.31

$Az_{\delta 1} = B \; or \; A = \frac{B}{z_{\delta 1}}$-------2.32

finally the open-loop TF is

$F_{\delta}(\gamma) = \frac{A\gamma + B}{\gamma[\gamma + (C-A)]}$ -----2.33

Methods of Controller designing: Consider the controller configuration of Figure 2.3. Let $P_{\delta}(\gamma)$ be the δ-domain equivalent of the continuous-time plant including the zero-order hold (ZOH), and $C_{\delta}(\gamma)$ be the transfer function (TF) of a rational cascade-controller, the parameters of which are to be determined. The δ-domain representation of the system is given in Figure 2.4. This design method is based on the frequency domain approximate model matching concept described above. The design objectives (i.e. the desired time and frequency domain specifications) are translated into a rational transfer function model. The controller parameters are then determined such that the closed-loop system with the above controller (Figure 2.4) approximates that of the specification model in some sense.

Table 2.1: Time domain specification for reference model

C/L sys. in - plane γ -plane				C/L step res		
p	$\omega\Delta$	Poles	Zeros	t_p/Δ	$\%M_P$	t_s/Δ
1	0.2999	-0.5759 + 0.4404i	-0.9007	7.8307	7.14507	10.1900
10	0.2999	-0.5759 + 0.4404i	-0.8042	7.3070	8.4616	10.2416
20	0.2999	-0.5759 + 0.4404i	-0.7140	6.7251	10.5225	10.4011
30	0.2999	-0.5759 + 0.4404i	-0.6332	6.1432	13.5479	10.6788
40	0.2999	-0.5759 + 0.4404i	-0.5560	5.5613	18.1738	11.0960
50	0.2999	-0.5759+ 0.4404i	-0.4775	4.9794	25.6998	11.6927
60	0.2999	-0.5759 + 0.4404i	-0.3926	4.3975	39.2036	12.5472
70	0.2999	-0.5759 + 0.4404i	-0.2943	3.8156	68.0051	13.8388
80	0.2999	-0.5759+ 0.4404i	-0.1710	3.2337	158.9355	16.1444
-1	0.2999	-0.5759 + 0.4404i	-0.9251	7.9471	6.9047	10.1901
-10	0.2999	-0.5759 + 0.4404i	-1.0549	8.4707	6.0098	10.2416
-20	0.2999	-0.5759 + 0.4404i	-1.2646	9.0526	5.3079	10.4011
-30	0.2999	-0.5759 + 0.4404i	-1.6341	9.6345	4.8538	10.6788
-40	0.2999	-0.5759 + 0.4404i	-2.5464	10.2164	4.6245	11.0961
-50	0.2999	-0.5759 + 0.4404i	-10.2796	10.7983	4.6446	11.6927
-60	0.2999	-0.5759 + 0.4404i	+2.8142	11.38025	5.0321	12.5472
-70	0.2999	-0.5759 + 0.4404i	+0.8292	11.9621	6.1997	13.8388
-80	0.2999	-0.5759 + 0.4404i	+0.2735	12.5441	10.2911	16.1444

Specifications for the desired performance of the closed-loop systems are formulated in the time domain (percentage overshoot, rise-time), frequency domain (gain margin, phase margin, resonant frequency etc.) or complex domain (damping ratio, frequency of damped oscillation) as discussed above. The closed loop reference model that satisfies a given set of desired performance specifications is shown in Figure 2.5 In the design method we use an open-loop-equivalent specification model $F_{\delta}(\gamma)$ as in Figure 2.6 of the closed-loop reference model (Figure 2.5), such that $F_{\delta}(\gamma)$ with unity-negative feedback equals $M_{\delta}(\gamma)$. So, for system in Figure 2.6 to be equivalent to the given closed-loop specification model in Figure 5.5 we have

$\frac{F_{\delta}(\gamma)}{1+F_{\delta}(\gamma)} = M_{\delta}(\gamma)$ ---2.35

Solving, we have

$F_{\delta}(\gamma) = \frac{M_{\delta}(\gamma)}{1-M_{\delta}(\gamma)}$ -----2.36

Table: 2.2 Coefficients of the reference model

ρ	Sys. coeff.		
	A	B	C
1	0. 5837	0.5257	1. 1519
10	0. 6536	0.5257	1. 1519
20	0. 7362	0.5257	1. 1519
30	0. 8302	0.5257	1. 1519
40	0. 9455	0.5257	1. 1519
50	1. 1008	0.5257	1. 1519
60	1. 3387	0.5257	1. 1519
70	1. 7859	0.5257	1. 1519
80	3. 0736	0.5257	1. 1519
-1	0. 5683	0.5257	1. 1519
-10	0. 4983	0.5257	1. 1519
-20	0. 4157	0.5257	1. 1519
-30	0. 3217	0.5257	1. 1519
-40	0. 2064	0.5257	1. 1519
-50	0. 0511	0.5257	1. 1519
-60	0. 1868	-0.5257	1. 1519
-70	0. 6340	-0.5257	1. 1519
-80	1. 92165	-0.5257	1. 1519

Thus given the desired specification model $M_\delta(\gamma)$, the open-loop equivalent specification model $F_\delta(\gamma)$ may be obtained as in equation (3.36). We choose a realizable discrete-time controller transfer function $C_\delta(\gamma)$ of order *q « n*.

We proposed one method for controller design of SISO systems in δ-domain. The first method is the extension of the continuous-time CPA technique in the δ- domain defined as DTM technique. The method presented yield controllers whose orders are known a-priori and the output of the controlled process matches that of the reference model.Now we will discuss some example in SISO system.

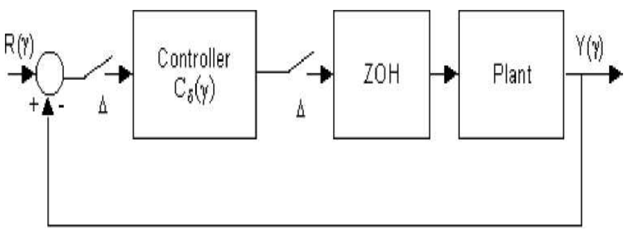


Figure 2.3: Unity negative feedback sampled-data control system

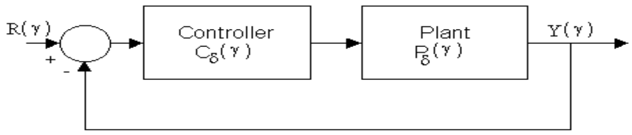


Figure: 2.4 the δ -domain representation of the system in Figure 2.3

PID Controller:

Example 2.1: For illustrating the methodology of the scheme, a simple plant is taken as:

$$G_p(s) = \frac{3}{s^2 + 4s + 3}$$

The following model transfer function satisfies: ω_n=5.0, ξ=0.707

$$M(s) = \frac{4.242s + 25}{s^2 + 7.07s + 25}$$

We choose a PID type of pre-compensator:

$$C(s) = k_P + \frac{k_I}{s} + k_D s$$

The parameters of the compensator, obtained by minimizing the H_∞ are given in Table 2.3.
In discrete delta domain taking delta=0.1

$$G_\delta(\gamma) = \frac{0.1315\gamma + 2.466}{\gamma^2 + 3.543\gamma + 2.466}$$

& $$M_\delta(\gamma) = \frac{3.898\gamma + 17.56}{\gamma^2 + 6.824\gamma + 17.56}$$

The parameters of the compensator obtained by model matching method using genetic algorithm are also given in the table2.3. The time response obtained by continuous time and discrete delta domain is shown in the fig.2.5

Table: 2.3 Controller parameters.

Fitness value	Range	Value obtained from continuous time using GA	Value obtained from discrete delta domain using GA	Paramter
0.0121	1,1.5	1.3612	1.3412	k_D
	9,11	10.0473	10.0675	k_p
	8,9	8.0576	8.0386	k_I

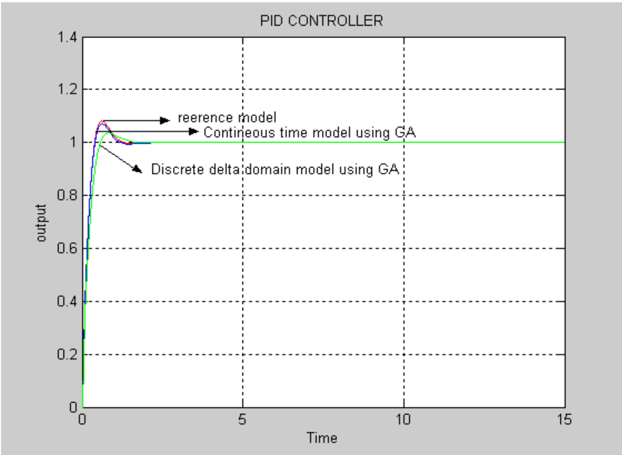


Fig: 2.5

Phase Lead Type Controller

Example 2.2: Let the plant be given by

$$G_p(s)\frac{20(1+\frac{s}{1.5})}{s(1+\frac{s}{4.5})(1+\frac{s}{10})(1+\frac{s}{30})}$$

The desired performance specifications are: velocity error constant K_v =20, damping ratio ξ=0.785, crossover frequency ω_c=4.5. Following the method of Chen and Shieh (1970), the following model is obtained:

$$M(s)=\frac{4.04265s+8.009}{s^2+4.4431s+8.009}$$

A phase-lead type of pre-compensator is chosen:

$$C(s)=\frac{k(s+\beta)}{(s+\alpha)}$$

The parameters of the controller obtained by using the proposed method are included in Table.2.4.

In discrete delta domain taking delta=0.1

$$G_\gamma(\gamma)=\frac{250.6\gamma^2+3466\gamma+4353}{\gamma^3+19.45\gamma^2+117.4\gamma+217.7}$$

and $M_\gamma(\gamma)=\dfrac{3.566\gamma+6.424}{\gamma^2+4.23\gamma+6.424}$

The parameters of the phase lead type of compensator obtained by model matching method using genetic algorithm are also given in the table 2.4. The time response obtained by continuous time and discrete delta domain is shown in the fig.2.6

Table: 2. 4 controller parameters.

Fitness value	Range	Value in continuous time	Value in Discrete Delta	
0.6654	0.07,0.1	0.0918	0.0901	k
	4,4.5	4.0053	4.000	β
	0.3,0.5	0.4138	0.4475	α

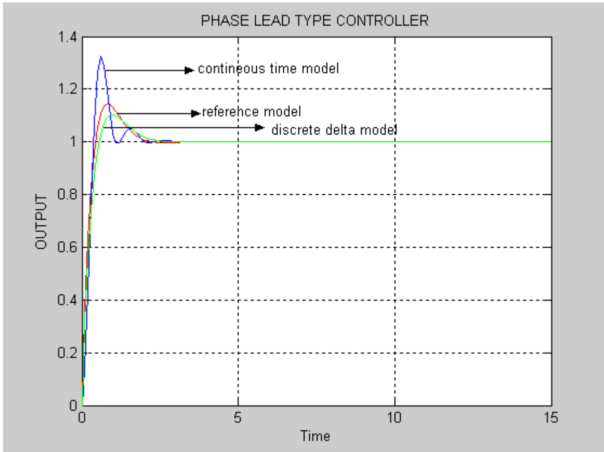


Fig: 2.6

Second Order Type Controller:

Example 2.3: Consider the following plant:

$$G_p(s)=\frac{20}{s(1+\frac{s}{10})(1+\frac{s}{30})}$$

The following desired model satisfies the values of velocity error constant K_v=20, damping ratio ξ=0.7, crossover frequency ω_c=5.

$$M(s)=\frac{4.35s+12.674}{s^2+4.984s+12.674}$$

We choose pre-compensator of the following structure:

$$C(s)=\frac{s^2+cs+d}{s^2+as+b}$$

The parameters of the controller obtained by using the proposed method are included in Table 2.5.

In discrete delta domain taking delta=0.1

$$G_\delta(\gamma)=\frac{94.61\gamma+1201}{\gamma^2+15.82\gamma+60.06}$$

and $M_\delta(\gamma)=\dfrac{3.889\gamma+9.876}{\gamma^2+4.913\gamma+9.876}$

The parameters of the second order type of compensator obtained by model matching method using genetic algorithm are also given in the table2.3. The time response obtained by continuous time and discrete delta domain is shown in the fig.2.7

Table 2.5 controller parameters.

Fitness value	Range	Value in continuous time	Value in Discrete Delta	Parameters
0.382	22, 23	22.1188	22.8157	a
	13,14	13.6543	13.6407	b
	6.5,7.5	7.1810	7.0009	c
	13,14	13.6687	13.0265	d

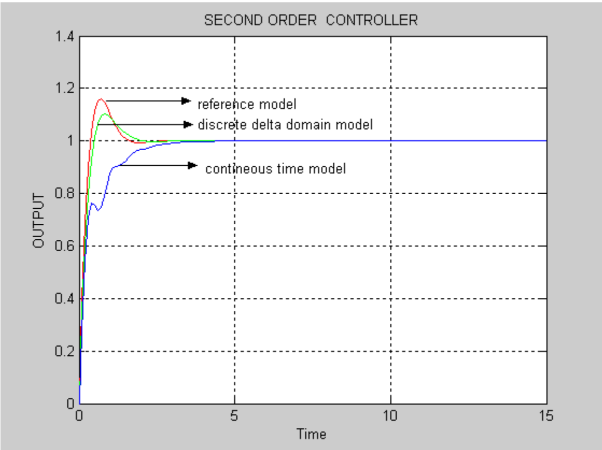


Fig: 2.7

Delay Type Controller:

Example 2.4: Consider a general second-order plant with time delay as: $G_p(s) = \dfrac{200e^{-s}}{s^2 + 10s + 100}$

We choose the same desired model as in Example 2.2, and incorporate a time –delay of 1sec, as shown below:

$$M(s) = \frac{(4.04265s + 8.009)e^{-s}}{s^2 + 4.4431s + 8.009}$$

We choose a PID controller as:

$$C(s) = k_P + \frac{k_I}{s} + k_D s$$

The parameters of the controller obtained by using the proposed method are included in Table 2.6. The time responses are given in Fig (2.8).

In discrete delta domain taking delta=0.1 & delay 1 sec.

$$G_\gamma(\gamma) = \frac{2.417\gamma^2 + 5.426\gamma + 3.045}{\gamma^4 + 4.443\gamma^3 + 7.403\gamma^2 + 5.482\gamma + 1.522}$$

$$M_\gamma(\gamma) = \frac{-8.882\text{e-}016\,\gamma^3 + 1.413\gamma^2 + 2.981\gamma + 1.555}{\gamma^4 + 4.447\gamma^3 + 7.437\gamma^2 + 5.545 + 1.555}$$

The parameters of the delay type of compensator obtained by model matching method using genetic algorithm are also given in the table2.6. The time response obtained by continuous time and discrete delta domain is shown in the fig.2.8.

Table: 2.6 controller parameters.

Fitness value	Range	Value in continuous time	Value in Discrete Delta	Parameters
0.7191	0.0003,0.005	0.0038	0.0040	k_D
	0.2,0.3	0.22993	0.2300	k_p
	0.3,0.6	0.48	0.4603	k_I

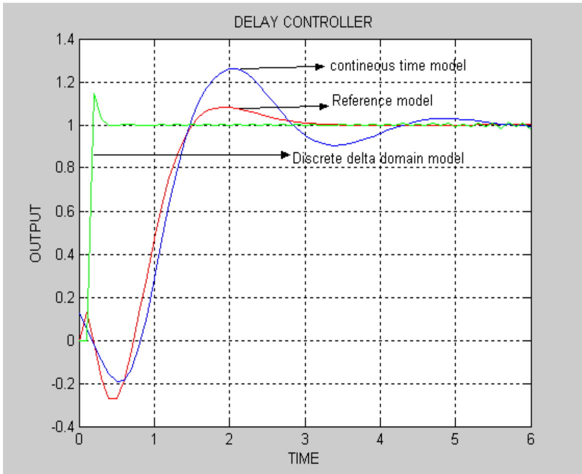


Fig: 2.8

3 Controller Design for MIMO Systems

Introduction: In recent years, considerable research efforts have been concentrated to develop time domain methods for design of controller based on state-space description. Although the state-space methods are computationally elegant, they require measurement of all the states leading to increased cost for control system design. In addition, if all the states are not available for measurement, we further require designing an observer to estimate the states. This complicates the structure of the control system and reduces the reliability of the overall system. In an alternative approach in this chapter, simple low-order dynamic controllers are designed that use only the available outputs for feedback purpose.

From a practical point of view, methods using only output feedback are normally preferred. A drawback of pole-placement techniques is that no zero-placement is done explicitly, while the transient response of a system depends very much on the positions of the poles as well as the zeros.

In model-matching type of controller design technique, a controller is designed such that the closed loop system behavior follows that of a reference model. The reference model is chosen to exhibit the desired transient and steady state responses. Methods based on exact model matching (EMM) often yield good matching at the cost of controller complexity. The resultant controllers may be of very high order and may sometimes not be realizable.

In this Paper three methods are proposed for designing cascade rational controllers for linear discrete-time multivariable industrial systems using output feedback. The methods are based on the principle of approximate model-matching as opposed to exact model-matching design procedures. Some illustrative examples are given to illustrate the usefulness of the proposed methods.

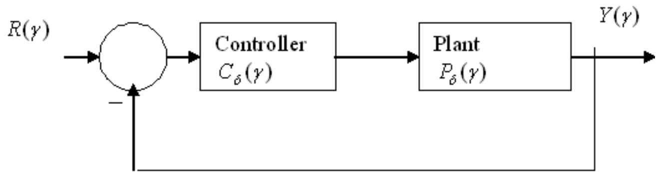


Fig: 3.1 Standard unity negative feedback configuration

MIMO controller design methods

The objective of the MIMO controller design methods based on the concept of AMM, is to find the controller TFM $C_\delta(\gamma)$ in Figure 1 such that the closed-loop system has satisfactory stability properties and the transient response to a specified demand vector r(t) follows closely that of the reference transfer function matrix $M_\delta(\gamma)$ of Figure 3.2. The precise design objectives and the degree of interaction permissible will, however, vary from application to application. The important general properties considered in the present work are

Stability
Closed-loop transient performance
Steady-state response and steady-state errors
Interaction minimization between various input-output loops

The MIMO controller design methods presented in this Paper are based on model matching technique. .

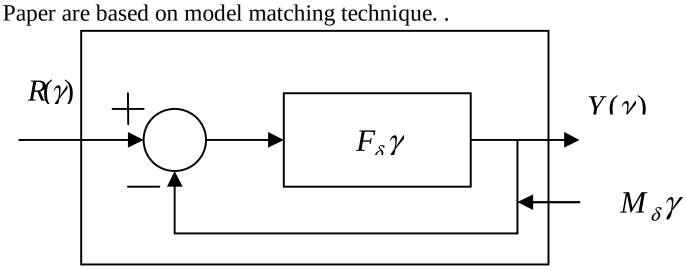


Fig: 3.2 the reference model for desired closed loop control system

MISO SUB OPTIMAL CONTROL SYSTEM

Example 1: We consider the voltage-regulator example from Sannuti and Kokotovic (1969):

$$A=\begin{bmatrix}-0.2 & 0.5 & 0 & 0 & 0\\ 0 & -0.5 & 1.6 & 0 & 0\\ 0 & 0 & -14.29 & 85.715 & 0\\ 0 & 0 & 0 & -25 & 75\\ 0 & 0 & 0 & 0 & 10\end{bmatrix};\quad B=\begin{bmatrix}0\\0\\0\\0\\30\end{bmatrix};\quad C=\begin{bmatrix}1 & 0 & 0 & 0 & 0\end{bmatrix};$$

$$Q=\mathrm{diag}(1,0,0,0,0);\quad r=1;\quad K_{optimal}=[0.9245\quad 0.1711\quad 0.0161\quad 0.0492\quad 0.2643];$$

$$K_{Pal}=\begin{bmatrix}0.9245221 & 0.1705645 & 0.0405198 & 0 & 0\end{bmatrix}$$

A suboptimal controller has to be designed by using the first three states for feedback purpose. We choose the following structure for the feedback matrix K.

$$K=[x(1)\quad x(2)\quad x(3)\quad 0\quad 0]$$

Considering the same problem in discrete delta domain taking delta-0.1

$$A_\delta\gamma=\begin{bmatrix}-0.1980 & 0.4828 & 0.0255 & 0.0466 & 0.0845\\ 0 & -0.4877 & 0.8257 & 2.0659 & 5.3298\\ 0 & 0 & -7.6045 & 12.6022 & 65.1929\\ 0 & 0 & 0 & -9.1792 & 14.2897\\ 0 & 0 & 0 & 0 & -6.3212\end{bmatrix}$$

$$B_\delta\gamma=\begin{bmatrix}0.0592\\ 5.0917\\ 101.5256\\ 39.7432\\ 18.9636\end{bmatrix},\quad C_\delta\gamma=\begin{bmatrix}1 & 0 & 0 & 0 & 0\end{bmatrix}\ \&\ D_\delta\gamma=0.$$

The parameters of the multi input single output compensator are obtained by model matching method using genetic algorithm are also given in the table3.1 The time response obtained by continuous time and discrete delta domain is shown in the fig.3.3

The controller parameters are given in Table 3.1, and the time-response comparisons are given in Fig 3.3.

Table 3.1 Controller parameters.

Fitness value	Range	Value in continuous time	Value in Discrete Delta	Parameters
0.2443	0.92,0.93	0.9218	0.9218	x(1)
	0.17,0.18	0.1701	0.1793	x(2)
	0.016,0.05	0.0497	0.0414	x(3)

The parameters of the multi input single output sub optimal compensator are obtained by model matching method using genetic algorithm are also given in the table3.1 The time response obtained by continuous time and discrete delta domain is shown in the fig.3.3

The controller parameters are given in Table 3.1, and the time-response comparisons are given in Fig 3.3.

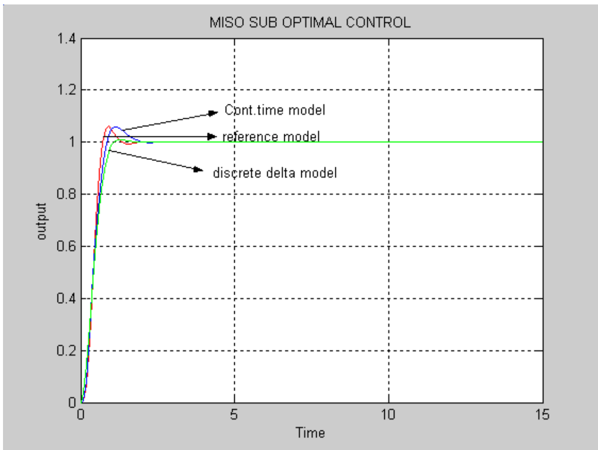


Fig: 3.3

MIMO SUB OPTIMAL COTROLSYSTEM

Example .2: The numerical data refers to a Mach2.7 flight condition of a supersonic transport aircraft [Markland, 1970].The system equations are:

$$A=\begin{bmatrix}-0.037 & 0.0123 & 0.00055 & -1\\ 0 & 0 & 0.08 & 0.804\\ -6.37 & 0 & -0.23 & 0.0618\\ 1.25 & 0 & 0.016 & -0.0457\end{bmatrix};\quad B=\begin{bmatrix}0.00084 & 0.000236\\ 0 & 0\\ 0.08 & 0.804\\ -0.0862 & -0.0665\end{bmatrix};$$

$$C=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix};\quad Q=r=I;\quad K_{optimal}=\begin{bmatrix}1.8623 & -0.1798 & -0.7008 & -6.4075\\ -3.9387 & 0.9279 & 1.5541 & 2.9926\end{bmatrix}$$

A suboptimal controller has to be designed by using the last three states for feedback purpose. We choose the following structure for the feedback matrix K.

$$K=\begin{bmatrix}0 & x(1) & x(2) & x(3)\\ 0 & x(4) & x(5) & x(6)\end{bmatrix}$$

The controller parameters are given in Table 3.2, and the time-response comparisons are given in Fig (3.4.)

Considering the same problem in discrete delta domain taking delta-0.1

$$A_\delta\gamma=\begin{bmatrix}-0.0993 & 0.0123 & 0.0004 & -0.9938\\ -0.3152 & -0.0001 & 0.9886 & 0.0136\\ -6.2686 & -0.0039 & -0.2275 & 0.3757\\ 1.2372 & 0.0008 & 0.0158 & -0.1075\end{bmatrix}$$

$$B_\delta\gamma=\begin{bmatrix}0.0051 & 0.0036\\ 0.0039 & 0.0399\\ 0.0777 & 0.7938\\ -0.0857 & -0.0656\end{bmatrix}\quad C_\delta\gamma=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

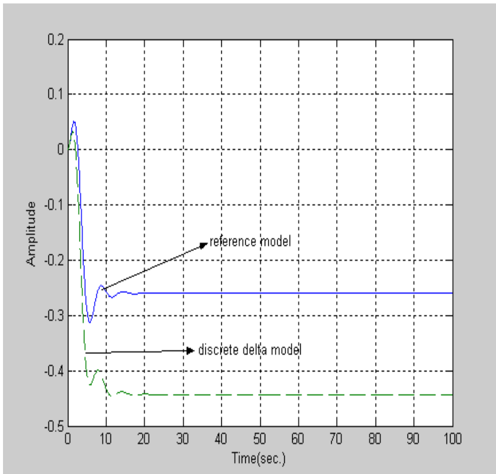
and $D_\delta\gamma=\begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}$

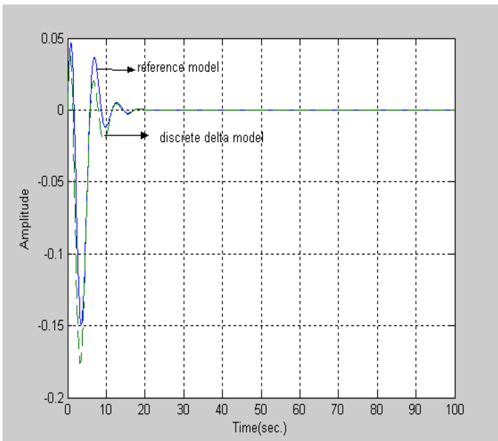
Table 3.2 Controller parameters.

Fitness value	Range	Value in continuous time	Value in Discrete Delta	Parameters
0.4617	-0.38, -0.35	-0.3689	-0.3688	x(1)
	-1.56, -1.5	-1.5297	-1.5297	x(2)
	-8, -6	-6.0248	-6.0254	x(3)
	1.2,1.5	1.2532	1.2725	x(4)
	3,4	3.0589	3.0276	x(5)
	4,6	5.4817	5.4818	x(6)

The parameters of the multi input multi output(MIMO) compensator are obtained by model matching method using genetic algorithm are also given in the table3.2The time response obtained by continuous time and discrete delta domain is shown in the fig.3.4

The controller parameters are given in Table 3.2 and the time-response comparisons are given in Fig3.4





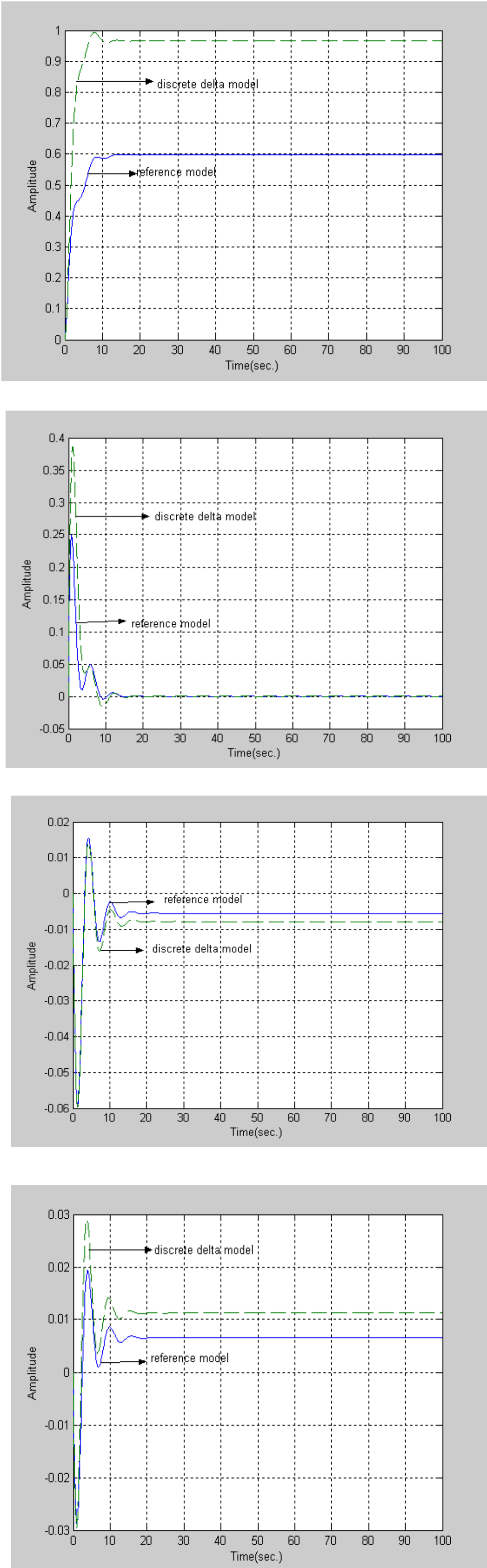


Fig: 3.4

Conclusions: The present work has dealt with the development control system design using of GA based techniques for solving various problems in single input and single output(SISO) control like, PID control, first order system control, second order system control, phase lead type control, multiple input and single output (MISO)control and multiple input and multiple output(MIMO) sub optimal control in continuous-time systems.

Overall, it may be stated that the proposed GA based methods for solving the above-mentioned control problems are viable and compare favorably with other established methods from the control literature. Throughout the paper several illustrative examples have been given to validate this claim, and for many examples, comparisons with other available results from the literature are included. The main advantages of the proposed techniques are in their general applicability, simplicity of the resultant controller / model, ease of formation of different fitness functions, no strict requirement of the initial guess vector, flexibility to work with fitness functions framed in the time or frequency domain, flexibility to work with a transfer function or state-space description and a general guarantee to arrive at (sub) optimal results.

In spite of the several problems considered in this work, many important areas in control like adaptive controller tuning, adaptive control, identification of multivariable nonlinear systems etc. may be taken up by future researchers.

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